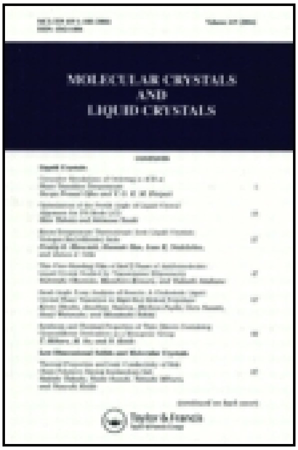

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Hydrodynamic Effects and Transient Transverse Fluctuations in The Freedericksz Transition

R. F. Rodríguez [a] , M. San Miguel [b] & F. Sagués [c]

[a] Departamento de Polimeros, Instituto de Investigaciones en Materiales, UNAM. A. P. 70-360, Coyoacán, México, D.F., 04510, Mexico

[b] Departament de Fisica, Universitat de les Illes, Balears.E-07071, Palma de Mallorca, Spain

[c] Departament de Quimica-Fisica, Universitat de Barcelona, Diagonal 647, 08028, Barcelona, Spain

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Hydrodynamic Effects and Transient Transverse Fluctuations in the Freedericksz Transition

R. F. RODRÍGUEZ

Departamento de Polímeros, Instituto de Investigaciones en Materiales, UNAM. A.P. 70-360, Coyoacán, México, D.F., 04510 México

and

M. San MIGUEL

Departament de Fisica, Universitat de les Illes Balears.E-07071 Palma de Mallorca, Spain

and

F. SAGUÉS

Departament de Química-Física, Universitat de Barcelona. Diagonal 647, 08028 Barcelona, Spain

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Hydrodynamic effects in the transient dynamics of the Freedericksz transition in the twist and homeotropic to planar geometries are analyzed. A novel dynamical anisotropic response for the transverse orientational fluctuations of the director at finite wave numbers is described.

Keywords: Freedericksz transition, nematics, hydrodynamic effects

The textbook analysis[1,2] of the transient dynamics associated with the Freedericksz transition considers a nematic sample spatially homogeneous in the (x,y) directions and contained between two plates perpendicular to the z-direction and separated a distance d. At these plates strong anchoring boundary conditions for the director are assumed. When the magnetic field is switched-on to a value $H > H_c$, the reorientational dynamics of the director is described by the evolution of the amplitudes of the discrete Fourier modes along z of an appropriate orientational angle.[3] The study of spatial inhomogeneities in the transverse (x,y) directions is also of interest for several reasons. First, even in the absence of hydrodynamic couplings, the transverse spatial fluctuations of the director field are anomalously large during the transient dynamics.[4] (By transverse spatial fluctuations we mean the correlated fluctuations of the director orientation occurring at different space points in the transverse (x,y) plane). Secondly, it is known that, for example in the twist geometry,[1,2] hydrodynamic couplings are responsible for the appearance of transient periodic spatial structures in one transverse direction when the magnetic field is large enough.[5,6] However, an analysis of possible periodicity in both x and y directions has not been reported. In this letter we consider the transient dynamics of the Freedericksz transition in the twist and homeotropic to planar geometries, allowing for transverse spatial inhomogeneities in both x and y directions and including hydrodynamic couplings. For a third possible geometry (planar to homeotropic transition), an $x - y$ periodicity has already been considered.[7] We address here two main questions. One concerns the possibility of the occurrence of two-dimensional transient spatial structures. The second question concerns the effect of the hydrodynamic couplings, when no transient spatial structures occur, in the transverse spatial fluctuations which are largely amplified during the transient dynamics. Our starting point for this analysis is the complete set of stochastic nematodynamic equations developed in Reference 6.

In the two geometries considered here the hydrodynamic coupling of the director with the velocity field leads to an effective viscosity coefficient γ for the director which depends on the wave number $q = (q_x,q_y)$. We make the usual assumption in which macroscopic flow only occurs along the direction of the applied magnetic field. In this case and in the twist geometry, we find a negative answer to the question of a possible transient pattern with periodicity in both x and y directions. We also find that no transient pattern occurs in the homeotropic to planar geometry.[8] On the other hand a rather interesting result is found in both geometries concerning hydrodynamic effects in the transient transverse spatial fluctuations. It happens that the effective viscosity coefficient is clearly anisotropic so that the transient amplification of the transverse spatial fluctuations has a largely different dependence on x and y. While the anisotropy of static properties of the nematic phase is well known this phenomenon appears as an example of novel dynamical anisotropic properties. The dynamical anisotropy appears as a result of hydrodynamic couplings involving the transverse response of the system at wave numbers $q \neq 0$.

Our general starting equations for the director $n(r,t)$ and velocity fields are[6]

$$d_t n_\beta = (1/\gamma_1)(\delta F/\delta n_\beta) + \Gamma_{\beta\gamma}(n)(\delta F/\delta v_\gamma) + \xi_\beta(r,t) \tag{1}$$

$$d_t \text{v}_\beta = L_{\beta\gamma}(n)(\delta F/\delta v_\gamma) - \Gamma^+_{\beta\gamma}(n)(\delta F/\delta n_\gamma) + \partial_\alpha \Omega_{\alpha\beta}(r,t) \tag{2}$$

where F is the free energy functional

$$F = (1/2)\int dr\, K_{\alpha\beta\gamma\delta}\, \partial_\beta n_\alpha\, \partial_\delta n_\gamma - (1/2)\int dr\, \chi_a\, (n_\alpha H_\alpha)^2 + (1/2)\int dr \rho\, \text{v}^2 \tag{3}$$

The first term gives the Oseen-Frank distortion free energy in terms of Frank's elastic constants K_1, K_2 and K_3. The second term is the magnetic free energy given in terms of the anisotropic part of the magnetic susceptibility χ_a and the magnetic field H_α. The third term gives the hydrodynamic contribution, being ρ the mass density. Equations (1) and (2) describe the coupled dynamics of n and v in terms of the free energy (3) and of the kinetic operations $\Gamma_{\beta\gamma}$ and $L_{\beta\gamma}$ which in turn are given[4] in terms of viscosity coefficients[1] γ_1, γ_2, ν_1, ν_2 and ν_3. The noise sources ξ_β and $\partial_\alpha\Omega_{\alpha\beta}$ are Gaussian white noise with zero mean. They satisfy fluctuation-dissipation relations such that the equilibrium distribution associated with (1), (2) has the canonical form $P_{eq}[n(r), \mathrm{v}(r)] \sim \exp[-F/k_B T]$:

$$\langle \xi_\beta(r,t)\xi_\gamma(r',t')\rangle = (2k_B T/\gamma_1)\delta(r - r')\delta(t - t')\,\delta_{\beta\gamma} \qquad (4)$$

$$\langle \partial_\alpha\Omega_{\alpha\beta}(r,t)[\partial_\delta\Omega_{\delta\gamma}(r',t')]^T\rangle = -2\,K_B T\,L_{\beta\gamma}\,\delta(r - r')\delta(t - t') \qquad (5)$$

In the absence of hydrodynamic couplings (1) reduces to the equation used to describe the fluctuating dynamics of the director[3,4]. In the absence of director dynamics (2) reproduces the fluctuating hydrodynamics of Laudau-Lifshitz.

We now specialize equations (1)-(3) to particular situations in the twist and homeotropic to planar geometries. In the two cases the sample is contained between two plates perpendicular to the z-axis located at $z = \pm d/2$. In the twist geometry the director is initally aligned along the x-axis ($n^0 = (1,0,0)$) and the magnetic field is applied along the y-axis. In the homeotropic to planar transition the director is initially along the z-axis ($n^0(0,0,1)$) and the magnetic field is applied along the x-axis. In both cases macroscopic flow is assumed to occur only in the direction of the applied field so that $v_x = v_z = 0$ in the twist case, and $v_y = v_z = 0$ in the homeotropic to planar transition. Also in both cases we keep the full dependence of the variables n and v in z and in the transverse coordinates (x,y). We invoke a minimal coupling approximation[6] in which the dependence of the kinetic operators on the director is approximated replacing n by n^0. In the twist geometry the director reorientation is assumed to take place in the x,y plane so that

$$n_x(r,t) = \cos \varnothing\,(r,t) \qquad n_y(r,t) = \sin \varnothing\,(r,t) \qquad (6)$$

and the stochastic equations become

$$d_t\varnothing(r,t) = (-1/\gamma_1)(\delta F/\delta\varnothing) + ((1 + \lambda)/2)\partial_x v_y(r,t) + \xi(r,t) \qquad (7)$$

$$d_t v_y(r,t) = ((1 + \lambda)/2\rho)\partial_x(\delta F/\delta\varnothing) + (1/\rho)(\nu_2\partial_z^{\,2} + \nu_3\partial_x^{\,2})V_y(r,t) + \partial_x\Omega_{xy} + \partial_z\Omega_{zy} \qquad (8)$$

where $\lambda = (\gamma_2/\gamma_1)$ and the incompressibility condition $\partial_y \mathrm{v}_y = 0$ has been used to simplify the systematic and stochastic contributions in (8). Likewise, in the homeotropic to planar geometry reorientation is assumed to take place in the x,z plane so that

$$n_x(r,t) = \sin \varnothing\,(r,t) \qquad n_z = \cos \varnothing\,(r,t)$$

The stochastic equations become

$$d_t\varnothing(r,t) = -(1/\gamma_1)(\delta F/\delta\varnothing) + ((1 + \lambda)2)\partial_z v_x(r,t) + \xi(r,t) \qquad (9)$$

$$d_t \mathrm{v}_x(r,t) = ((1 + \lambda)/2\rho)\partial_z(\delta F/\delta\varnothing) + (1/\rho)(\nu_2\partial_y^2 + \nu_3\partial_z^2)v_x(r,t) + \partial_y\Omega_{yx} + \partial_z\Omega_{zx} \qquad (10)$$

To proceed further we make the approximation of negligible inertia in which the velocity follows instantaneously the director dynamics: $d_t v_x = 0$. The elimination of the velocity leads to a closed equation for the angle $\emptyset$. This elimination is conveniently done using a Fourier transform. With $\rho = (x,y)$

$$\emptyset(r,t) = \Sigma_m \Sigma_q \Theta_{m,q}(t)\cos[(2m+1)(\pi z/d)]e^{iq\rho} \tag{11}$$

$$v(r,t) = \Sigma_m \Sigma_q \, v_{m,q}(t)\cos[(2m+1)(\pi z/d)]e^{iq\rho} \tag{12a}$$

$$v(r,t) = \Sigma_m \Sigma_q \, v_{m,q}(t)\sin[(2m+1)(\pi z/d)]e^{iq\rho} \tag{12b}$$

where (12a) is for the twist situation and (12b) for homeotropic to planar. Similar transformations are used for the noise sources. The index m numbers the discrete modes in the z-direction and q is the continuous wave number in the transverse plane. In the linear approximation the resulting equation for the amplitude $\Theta_{mq}(t)$ turns out to be:

$$\partial_t \Theta_{m,q}(t) = (1/\gamma'_m(q) W_m(q))\Theta_{m,q}(t) + \eta_{m,q}(t) \tag{13}$$

where $\gamma'_m(q)$ is an effective, wave number dependent, viscosity coefficient. The effect of the hydrodynamic coupling in the dynamics is contained in this effective viscosity. $W_m(q)/\gamma'_m(q)$ is the amplification factor of the fluctuations during the transient dynamics associated with the switching-on of the magnetic field. The noise source $\eta_{m,q}(t)$ is a linear combination of the Fourier amplitudes of ξ and $\partial_\alpha \Omega_{\alpha\beta}$. It satisfies a fluctuation dissipation relation with the effective viscosity

$$\langle \eta_{m,q}(t)\, \eta^*_{n,q'}(t')\rangle = 2(2k_\beta T/\gamma' V)\delta_{m,n}\, \delta_{q,-q'}\delta(t-t') \tag{14}$$

Where V is the volume of the system. The explicit forms of γ' and W are, for the twist geometry

$$\gamma'_m{}^T(q) \equiv \gamma_1 \Gamma_m{}^T(q_x) \equiv \gamma_1(1 - \alpha_2^2/\gamma_1(\eta_c + \nu_2\,(2m+1)^2\pi^2/d^2 q_x{}^2)) \tag{15}$$

$$W_m^T(q) = \chi_a H^2 - K_1 q_y^2 - K_3 q_x^2 - K_2(2m+1)^2\, \pi^2/d^2 \tag{16}$$

where $\alpha_2 = (1/2)\gamma_1(1+\lambda)$ and $\eta_c = \nu_3 + (\alpha_2^2/\gamma_1)$. For the homeotropic to planar geometry to find

$$\gamma'_m{}^{HP}(q) \equiv \gamma_1 \Gamma_m{}^{HP}(q_y) \equiv \gamma_1(1 - \alpha_2^2/\gamma_1(\eta_c + \nu_2\, q_y^2 d^2/(2m+1)^2\pi^2) \tag{17}$$

$$W_m{}^{HP}(q) = \chi_a H^2 - K_1 q_x^2 - K_2 q_y^2 - K_3(2m+1)^2\pi^2/d^2 \tag{18}$$

The questions addressed in our introductory remarks can be now answered in view of Equations (13), and (15–18). In fact, the important point is the wave-number dependence of the effective viscosity coefficients. We note that $\gamma'_m{}^T(q=0) = \gamma_1$ so that the hydrodynamic coupling disappears for homogeneous reorientation in the twist geometry, while an effective viscosity still exists[2,8] for $q = 0$ in the HP geometry. The instability of a given mode is determined by the positivity of $W_m(q)$: There exists a range of q- modes associated with $m = 0$ which become unstable when $H > H_c$ with $H^2_{c,T} = (K_2\pi^2/\chi_a d^2)$ and $H^2_{c,\mathrm{HP}} = (K_3\pi^2/\chi_a d^2)$. A transient spatial pattern associated with the mode $m = 0$ with characteristic periodicity q^0 appears when the product $\gamma_0'^{-1}(q)\ W_0(q)$ has a maximum at $q = q^0 \neq 0$ within the range of unstable q-modes. For the twist geometry such transient pattern occurs for large enough applied-fields.[6] However $\gamma_m'^{\ T}(q)$ does not depend on q_y so that a pattern only occurs in the x-direction in spite of the systematic inclusion of inhomogeneities in the y-direction considered in this paper. These inhomogeneities were absent in the analysis of Ref 6. In the HP geometry contrary to what happens in the twist geometry, $\gamma_m'^{\mathrm{HP}}\ (q)$ is a monotonous increasing function of q so that the maximum of $\gamma_m'^{\mathrm{HP}}\ (q)^{-1}\ W_m^{\ \mathrm{HP}}(q)$ occurs at $q = 0$ and no transient pattern ever occurs.

We next consider the transient amplification of transverse spatial fluctuations in the situations just described in which spatial patterns do not occur. These fluctuations are described by the time dependent structure factor $C_{m,q}(t) \equiv \langle\Theta_{m,q}(t)\,\Theta_{m,-q}(t)\rangle$. Introducing magnetic coherence lengths $\rho_i = K_i/\chi_a\ H^2$ and a new time scale $\tau = [1\text{-}(H_c/H)^2]\chi_a\ H^2\ t/\gamma_1$, we find

$$\partial_\tau C_{m,q}(\tau) = (2/\Gamma_m(q))(1 - Q^2/q_c^2)\ C_{m,q}(\tau) + (2/\Gamma_m(q))\varepsilon \tag{19}$$

where $\Gamma_m(q)$ is given by (15) and (17) in the two respective geometries and $\varepsilon = 2\ K_BT/\{V\ \chi_a H^2\ [1\text{-}(H_c/H)^2]\}$. The anisotropy averaged transverse wave vector Q is in the two geometries considered

$$Q_T^2 = (\rho_1^2/\rho_2^2)\ q_y^2 + (\rho_3^2/\rho_2^2)q_x^2 \tag{20}$$

$$Q_{\mathrm{HP}}^2 = (\rho_1^2/\rho_3^2)q_x^2 + (\rho_2^2/\rho_3^2)q_y^2 \tag{21}$$

q_c^2 fixes the range of unstable q-modes. In the twist geometry

$$q_c^2 = \rho_2^{-2}\,[1 - (2m + 1)^2\ (H_c/H)^2] \tag{22}$$

and the same expression in the HP geometry with ρ_2 replaced by ρ_3. Equation (19) is of the same form than the one studied in Reference 4 except for the q-dependence of $\Gamma_m(q)$. That analysis neglected hydrodynamic coupling. Much of the discussion of that case applies here. We next focus on the specific consequences of the presence of the q-dependent viscosity coefficient.

The amplification factor in (19) contains two well differentiated factors. The factor $(1 - Q^2/q_c^2)$ comes from the form of the free energy. It is independent of the dynamics and therefore indepenent of the hydrodynamic coupling. The factor $\Gamma_m^{\ -1}(q)$ contains the dynamic effects originated in the hydrodynamic couplings. The statics of the nematic state show certain anisotropies for $K_1 \neq K_2 \neq K_3$. Such anisotropy in the free energy can be hidden by appropriate space rescaling of the transverse coordinates. This is the contents of (20)-(21) which permit to express the static part coming from the free energy of (19) in terms of a redefined wave-number Q^2. A dynamic anisotropy also appears in (19) due to the fact that $\Gamma_m(q)$ has not a symmetric dependence on q_x and q_y. This anisotropy cannot be overcome by a wave-number independent change of time scale. It is reflected in the solution of (19) in the fact that the amplification rate of the transverse fluctuations in the x and y direction is different between themselves and different in the two geometries considered. We note that this difference only appears during the transient state since equilibrium fluctuations are independent of $\Gamma_m(q)$. In particular, the solution of (19) involves[4] an initial condition which is independent of $\Gamma_m(q)$. This is so because the initial condition corresponds to the equilibrium fluctuations at $t = 0$ with an applied field $H < H_c$. The coefficient $\Gamma_m(q)$ determines the rate of initial amplification of fluctuations and also the rate of approach to the final equilibrium state. Therefore, for example for the HP geometry, the transverse spatial fluctuations in the x-direction will initially grow much faster than the ones in the y-direction and they will also decay at a slower rate. Thus, hydrodynamics reduce the transient anomalous fluctuations in the y-direction. Corresponding comments apply to the twist geometry.

We finally note that the qualitative features of the dynamical anisotropies just discussed are not changed by the consideration of nonlinear contributions.[9] Nonlinear terms neglected in (19) can be handled by a Gaussian decoupling scheme as in Reference 4. In this scheme the nonlinear term is approximated by a decoupling in which the nonlinearity appears effectively at $q = 0$. In this approximation the nonlinearity does not introduce any new anisotropy.

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